Useful Integrals

$$\int_0^\infty x^n \cdot e^{-b \cdot x}\, dx = \frac{n!}{b^{n+1}}$$

Version with non-infinite limit..

$$\int x \cdot e^{a \cdot x}\, dx = \frac{e^{a \cdot x}}{a^2} \cdot \left(e^{a \cdot x} - 1\right)$$

$$\int_0^\infty e^{-b \cdot x^2}\, dx = \frac{1}{2} \cdot \sqrt{\frac{\pi}{b}}$$

$$\int_0^\infty x^{2n+1} \cdot e^{-b \cdot x^2}\, dx = \frac{n!}{2 \cdot b^{n+1}}$$

$$\int_0^\infty x^{2n} \cdot e^{-b \cdot x^2}\, dx = \frac{1 \cdot 3 \cdot 5 .. 2n - 1}{2^{n+1} \cdot b^n} \cdot \sqrt{\frac{\pi}{b}}$$

NOTE:

Integrals having symmetric limits..

If integrand is odd, integral = 0

If integrand is even, then

$$\int_{-b}^{b} \text{even\_integrand}\, dx = 2 \cdot \int_0^b \text{even\_integrand}\, dx$$

$$\int_0^c e^{-b \cdot x^2}\, dx = \frac{1}{2} \cdot \sqrt{\frac{\pi}{b}} \cdot \operatorname{erf}\left(c \cdot \sqrt{b}\right)$$

$$\int_c^\infty e^{-b \cdot x^2}\, dx = \frac{1}{2} \cdot \sqrt{\frac{\pi}{b}} \cdot \operatorname{erfc}\left(c \cdot \sqrt{b}\right)$$

erf = "error function"
erfc = "error function compliment"

Note: erf(x) + erfc(x) = 1

$$\int \sin(bx)^2\, dx = \frac{x}{2} - \frac{1}{4 \cdot b} \cdot \sin(2bx)$$

$$\int x \cdot (\sin(bx))^2\, dx = \frac{x^2}{4} - \frac{x \cdot \sin(2bx)}{4b} - \frac{\cos(2bx)}{8 \cdot b^2}$$

$$\int x^2 \cdot (\sin(bx))^2\, dx = \frac{x^3}{6} - \left(\frac{x^2}{4 \cdot b} - \frac{1}{8 \cdot b^3}\right) \cdot \sin(2bx) - \frac{x \cdot \cos(2bx)}{4 \cdot b^2}$$

$$\int \cos(bx)^2\,dx = \frac{x}{2} + \frac{1}{4\cdot b}\cdot\sin(2bx)$$

$$\int x\cdot\cos(bx)^2\,dx = \frac{x^2}{4} + \frac{x\cdot\sin(2bx)}{4b} + \frac{\cos(2bx)}{8\cdot b^2}$$

$$\int x^2\cdot(\cos(bx))^2\,dx = \frac{x^3}{6} + \left(\frac{x^2}{4\cdot b} - \frac{1}{8\cdot b^3}\right)\cdot\sin(2bx) + \frac{x\cdot\cos(2bx)}{4\cdot b^2}$$

$$\int \cos(b\cdot x)\cdot\sin(b\cdot x)\,dx = \frac{1}{2\cdot b}\cdot\sin(b\cdot x)^2$$

$$\int \cos(m\cdot x)\cdot\sin(n\cdot x)\,dx = \frac{\cos[(m-n)\cdot x]}{2\cdot(m-n)} - \frac{\cos[(m+n)\cdot x]}{2\cdot(m+n)} \qquad \text{for} \quad m^2 \neq n^2$$

$$\int \sin(m\cdot x)\cdot\sin(n\cdot x)\,dx = \frac{\sin[(m-n)\cdot x]}{2\cdot(m-n)} - \frac{\sin[(m+n)\cdot x]}{2\cdot(m+n)} \qquad \text{for} \quad m^2 \neq n^2$$

Common Integrals in kinetics..

$$\int \frac{1}{(a+bx)\cdot(c+dx)}\,dx = \frac{1}{ad-bc}\cdot\ln\left(\frac{c+dx}{a+bx}\right)$$

$$\int \frac{1}{z^n\cdot t^m}\,dx = \frac{1}{(n-1)\cdot\Delta}\cdot\frac{1}{t^{m-1}\cdot z^{n-1}} + \frac{(m+n-2)\cdot d}{(n-1)\cdot\Delta}\cdot\int \frac{1}{z^{n-1}\cdot t^m}\,dx$$

or...

$$\int \frac{1}{z^n\cdot t^m}\,dx = \frac{-1}{(m-1)\cdot\Delta}\cdot\frac{1}{t^{m-1}\cdot z^{n-1}} + \frac{(m+n-2)\cdot b}{(m-1)\cdot\Delta}\cdot\int \frac{1}{z^n\cdot t^{m-1}}\,dx$$

where.. $z = b + bx$ $\qquad$ $t = c + dx$ $\qquad$ and $\qquad$ $\Delta = bd - bc$