

Useful Integrals

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^{\infty} e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}$$

$$\int_0^{\infty} x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

$$\int_0^{\infty} x^{2n} e^{-ax^2} dx = \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}}$$

$$2 \cdot \sqrt{\frac{a}{\pi}} \cdot \int_0^b e^{-ax^2} dx = \operatorname{erf}(b\sqrt{a})$$

$$2 \cdot \sqrt{\frac{a}{\pi}} \cdot \int_b^{\infty} e^{-ax^2} dx = \operatorname{erfc}(b\sqrt{a})$$

Note: $\operatorname{erf}(x) + \operatorname{erfc}(x) = 1$

$$\int \frac{dx}{(a+bx)(c+dx)} = \frac{1}{ad-bc} \ln\left(\frac{c+dx}{a+bx}\right)$$

$$\begin{aligned} \int \frac{dx}{z^n t^m} &= \frac{1}{(n-1) \cdot \Delta} \cdot \frac{1}{t^{m-1} z^{n-1}} + \frac{(m+n-2) \cdot \beta}{(n-1) \cdot \Delta} \int \frac{dx}{z^{n-1} t^m} \\ &= \frac{-1}{(m-1) \cdot \Delta} \cdot \frac{1}{t^{m-1} z^{n-1}} - \frac{(m+n-2) \cdot b}{(m-1) \cdot \Delta} \int \frac{dx}{z^n t^{m-1}} \end{aligned}$$

where $z = a + bx$, $t = \alpha + \beta x$ and $\Delta = a\beta - \alpha b$